Dynamics of Willapa Bay, Washington, a highly unsteady partially mixed estuary: 
II. Comparing tide- and density-driven exchange

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Abstract

Willapa Bay is a small, macrotidal estuary subject to large variations in both river input and ocean salinity. As a result, the net gain or loss of salt is frequently a first-order contributor to the estuarine salt balance on both subtidal and seasonal timescales, as discussed in a companion paper in this volume by Banas, Hickey, and Newton. It is shown here that in such an unsteady salt balance, the classification scheme of Hansen and Rattray 1966 dramatically underestimates the role of salt-flux mechanisms other than the gravitational circulation. A horizontal diffusivity parameterizing the non-gravitational ("diffusive") salt flux is determined empirically for Willapa, by finding the slope between the rate of change of salt storage and the along-channel salinity gradient in low-riverflow conditions. This diffusivity varies along the length of the estuary as 10–20% of the rms tidal velocity times channel width, consistent with density-independent dispersion by tidal residuals tied to bathymetry. Diffusive salt flux is compared with gravitational-circulation salt flux, and is found to dominate over most of the channel except at the highest flows. This direct assessment of the diffusive salt-flux fraction, and of the total ocean-estuary exchange rate, departs from the predictions of the Hansen-and-Rattray steady-salt-balance theory by an order of magnitude.

1. Introduction

This is the second of two papers in this volume (see Banas et al. 2002, hereafter BHN02) examining the dynamics of Willapa Bay, Washington, USA on subtidal to seasonal timescales. The previous paper described the relative roles, under variations in riverflow and ocean salinity, of three of the four components of the subtidal, whole-estuary salt balance: net changes in salt storage; seaward, river-driven advection of salt;
and density-driven exchange with the ocean. In this paper we determine the role of the fourth component—tide-driven, or diffusive, exchange—relative to density-driven exchange.

Willapa is a member of a system of small estuaries on the Pacific Northwest coast (Emmett et al. 2002) that experience large (> 2 m) tidal ranges, and event-scale (2-10 d) ocean water-property changes up to 10 psu and 5°C. These changes are forced by wind-driven upwelling and downwelling and (on the Washington coast) also by episodic intrusions of the Columbia River plume (Hickey and Banas 2002). These estuaries have highly seasonal riverflow that is negligible for most of the summer, though often strong and dynamically important (BHN02) during winter. They also tend to have broad, shallow intertidal zones that constitute a large fraction of total area and volume (Hickey and Banas 2002).

BHN02 show that the cross-sectionally averaged salt balance in Willapa is frequently unsteady to lowest order even at very long (> 50 d) timescales: that is, net fluxes of salt in or out of the estuary often exceed river-driven, seaward advection of salt ("river-flushing") by an order of magnitude or more. Despite this result, they find that the vertical density-driven ("gravitational") circulation scales directly with riverflow. That is, baroclinic loading of salt never exceeds river-flushing by more than a factor of ~2, even when river-flushing is negligible in the overall salt balance.

In such conditions, the salt balance cannot close unless a non-gravitational mechanism is involved in ocean-estuary exchange. A description of this process—its magnitude relative to the exchange flow, and its explanation in terms of tidal processes—is the subject of this paper. We will show that despite the direct relationship between river input and hydrography, and even in highly stratified conditions, the largest contributor to ocean-estuary exchange in the average over several forcing events appears to be density-independent dispersion by lateral tidal circulations.
More generally, we are concerned with the applicability of a widely-used diagnostic and classification scheme, that of Hansen and Rattray 1965, 1966 (hereafter HR65, 66), to estuaries in which unsteady salt fluxes are large. Such unsteadiness may be driven by riverflow, local wind, sea level, tidal, or external water-property variability, as reviewed by BHN02. Note also that unsteady dynamics may be important at short timescales (below the estuarine "adjustment time": Kranenburg 1986, MacCready 1999) even if a steady theory holds at longer timescales.

The HR66 description of partially mixed estuaries relies on a balance of three fluxes: down-estuary advection of salt by the mean flow, up-estuary return of salt by the gravitational circulation, and an up-estuary "diffusive" flux, representing all other exchange, which closes the balance. Estuaries can then be categorized by a parameter $\nu$, the "diffusive fraction of up-estuary salt flux," which might better be called the "nongravitational fraction" (Fischer 1976). Estuaries with $\nu = 0$ are tidally driven and unstratified, while estuaries with $0 < \nu < 1$ are partially-mixed. Since the sum of diffusive and gravitational up-estuary fluxes is constrained by river output in this balance, $\nu$ simultaneously represents (1) the strength of diffusive (density-independent) flushing mechanisms, and (2) the response of the gravitational circulation to riverflow.

Without the constraint of a steady salt balance, there need not be any relationship between these two processes. The steadiness of an estuarine salt balance, however, cannot be tested unless each term in the decomposition of upstream salt flux is determined individually and empirically, and this has been done in relatively few systems (Winterwerp 1983, Lewis and Lewis 1983, Dronkers and van de 1986, Simpson et al. 2001). More often, nongravitational salt flux is inferred from the other salt-balance terms (e.g. Oey 1984), or else all up-estuary salt fluxes are combined and represented by a single dispersion process (e.g., Uncles and Stephens 1990, Monismith et al. 2002). Thus $\nu$ is rarely actually calculated from data by summing up-estuary salt fluxes and finding
the diffusive fraction, as its name suggests. When \( \nu \) is not determined directly in this way, its accuracy can only be as good as the steady-state assumption used to define it.

Jay and Smith (1990) note that the subtidal mean flow is not, as HR65 assumes, separable from time-dependent tidal processes, but rather directly generated by those processes: strain-induced stratification variations between flood and ebb, for example. Thus the mean subtidal quantities by which HR65 and HR66 characterize an estuary might not represent a state ever realized by the estuary. This prompts Jay and Smith to propose an alternative estuarine classification scheme, based on the nonlinearities in the tidal salt balance. The example of Willapa suggests a parallel concern at a longer timescale horizon. Just as a subtidal average may describe conditions never realized at any point in the tidal cycle, so may the long-term average required by the HR66 scheme describe a steady balance never realized at subtidal timescales of interest. We will show that for much of the year in Willapa, the three terms in the Hansen and Rattray salt budget do not balance even to order of magnitude at any timescale from days to months.

In section 2 we summarize our program of observations and review the formulation of the salt balance derived by BHN02. Section 3 considers the magnitude of nongravitational (tidal) exchange, diagnosed from the low-riverflow-period salt balance. Section 4 compares tidal exchange to the gravitational circulation, and shows that the results differ dramatically from the predictions of the HR66 theory.

2. Data and calculations

In this section we describe the study area and dataset, present a convenient form of the volume-averaged, subtidal salt balance, and give methods of calculation for each quantity in that balance. This is largely a summary of results from BHN02, which calculated all terms in the salt balance except the tidal diffusive term: only essential information is repeated here.
We have confined our attention to Stanley Channel, one of three main branches of Willapa Bay (Fig. 1). This channel is 10-20 m deep and is surrounded by extensive tidal flats. Mean tidal range is ~ 2 m, and varies only 20% over the length of the estuary; the tidal excursion distance is 12-15 km. The Naselle River, which supplies ~20% of total river input, forms the head of Stanley Channel, 35 km from the mouth. The largest rivers, the Willapa and the North, lie closer to the mouth.

This study uses salinity data from August 1997 to February 2001 from five mooring stations and from 54 hydrographic transects of Stanley Channel during that period (Fig. 1). W3 and W6 are taut-wire moorings instrumented in the mid-to-lower water column; Bay Center, Oysterville, and Naselle are piling-mounted packages that follow sea level 1 m below the surface. W6 lies a short distance outside Stanley Channel and in addition the salinity record there is relatively short (~ 6 months), and so we have excluded this station from most of the analysis. Mooring salinities are from a combination of SeaBird Microcat CT sensors and Aanderaa current meters, which carry CT cells. Transect data are from SeaBird 19 and 25 CTDs.

Single-point, lower-water-column velocity data from W3 and W6 are also used to regress exchange-flow shear velocity to axial salinity gradient (sec. 4c,d,e: see Hickey et al. 2002, BHN02). Other auxiliary data sources include riverflow time series from USGS gauges 12010000 and 12013500 on the Naselle and Willapa Rivers, NOAA tidal-height predictions for Toke Point and Nahcotta (stations 9440910 and 9440747), and bathymetry data from a 1999 Army Corps of Engineers survey (Kraus 2000).
b. Calculation of the subtidal salt balance

By volume-averaging the subtidal tracer equation for salt from far upstream to an arbitrary cross-section, BHN02 derive the salt balance

\[ l_a \frac{\partial \bar{s}}{\partial t} + \frac{Q}{a} \bar{s} = u_e \delta s + K \frac{\partial \bar{s}}{\partial x} \]  

where

- \( x \) is along-channel (axial) distance
- \( t \) is time
- \( a \) is cross-sectional area
- \( l_a \) is a scale length, upstream volume divided by \( a \)
- \( Q \) is river volume flux
- \( \bar{s} \) is cross-sectionally-averaged salinity
- \( \bar{\bar{s}} \) is volume-averaged salinity upstream of \( x \)
- \( \delta s \) is a vertical salinity difference, or "stratification"
- \( u_e \) is the velocity scale of the baroclinic shear flow
- \( K \) is a horizontal diffusivity parameterizing all salt fluxes besides the mean vertical exchange flow.

We can interpret these four terms as (i) the rate of change of upstream salt storage, or time-change salt flux; (ii) downstream advection of salt by the mean flow, or river-flushing salt flux; (iii) the gravitational circulation; and (iv) diffusive salt flux, as discussed in detail in section 3. In this notation, the steady salt balance assumed by HR66 is simply

\[ \frac{Q}{a} \bar{s} = u_e \delta s + K \frac{\partial \bar{s}}{\partial x} \]  

The gravitational term \( u_e \delta s \) is more precisely expressed as a correlation in the cross-sectional average \( \overline{u' \bar{s}} \), where \( u' \) and \( s' \) are vertical variations in velocity and salinity around their cross-sectional means (\( \bar{u} = Q/a \) and \( \bar{s} \)). We have written this term
as the product of a scale shear velocity $u_e$ and a vertical salinity difference $\delta s$ since this is the level of precision at which our dataset allows us to calculate it. Note also that since mooring data are single-depth, $\delta s$ must be determined from instantaneous transects, which are tidally aliased. During a June 2000 experiment cited by BHN02, $\delta s$ varied by roughly a factor of two over one flood tide.

We use bay-total riverflow for $Q$ at W3, Bay Center, and Oysterville, to reflect the influence of the large rivers outside Stanley Channel, and use Naselle flow alone at the Naselle station. Particularly at Oysterville, the use of bay-total flow may be an overestimate by a factor of 2 or 3. Data sources and methods of calculation for the other quantities in (1) are summarized in Fig. 2. BHN02 give details for these methods and estimate the associated errors: they find that the limiting uncertainty in the calculation of term ratios is the approximately factor-of-two uncertainty in stratification caused by tidal aliasing of the transect observations. Note that BHN02 do not consider the diffusivity $K$ or the tidal-stirring term in (1): the interpretation and estimation of $K$ is the subject of the next section.

3. Tidal stirring and diffusive salt flux

a. Theory

There are any number of simultaneous horizontal "diffusion" processes in a partially mixed estuary like Willapa: most notably, vertical shear dispersion, wind-driven flows, and complex, three-dimensional density-driven circulations, in addition to lateral tidal asymmetries and nonlinearities. (Fischer 1976 and Zimmerman 1986 offer good reviews of the variety of lateral dispersion mechanisms.) In this context—on whole-estuary rather than turbulent scales—"diffusion" really means "complex advection," or "advection too intricate in space and time to treat in any other terms."
Thus to the list of diffusion processes above we might add the gravitational circulation itself, which recent studies are coming to portray not as a simple, steady advection cell but rather as a process episodic in time and anisotropic and evolving in the cross-channel direction (Jay and Smith 1988, Stacey et al. 2001).

Zimmerman (1986) offers two models to explain the large ($O(100-1000 \, m^2 \, s^{-1})$) diffusivities commonly observed in tidal waters. The first consists of a cascade of turbulent processes, in which lateral shears amplify dispersion by vertical shears which in turn amplify dispersion by small-scale turbulence proper. The second relies on lateral shears alone. He proposes dispersion by "Lagrangian chaos," in which the tidally averaged flow field is deterministic and steady but spatially complex—randomized by bathymetry rather than by turbulence—so that Lagrangian trajectories through that flow field are strongly divergent. Complex tidal residual flow fields are often observed (Kuo et al. 1990, Li and O'Donnell 1997, Blanton and Andrade 2001) and the divergence of trajectories in a such a field is clear from simulation (Ridderinkhof and Zimmerman 1992). The open question is how commonly this type of density-independent tidal dispersion dominates over density-driven mechanisms, or interactions between tidal and density-driven mechanisms (Smith 1996, Valle-Levinson and O'Donnell 1996), in estuaries with measurable stratification.

MacCready (1999) suggests that in shallow estuaries, horizontal diffusivity may indeed scale primarily with the amplitude and width of the largest lateral tidal residual circulations. In a small, macrotidal system like Willapa, in which the tidal excursion is larger than the width of the estuary, this model implies a scaling

$$K \approx c_K U_T b$$

(3)

where $c_K$ is a constant of proportionality $O(1)$ or smaller, $U_T$ is the rms tidal velocity (usually several times larger, at least, than the subtidal velocity field $u$) and $b$ is the channel width. On the principle that tidal residual velocities tend to be 10-20% of the tidal amplitude (Zimmerman 1986), we can estimate $c_K \sim 0.1–0.2$. 

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A scaling law like (3) in which wind stress replaces tidal velocity could potentially hold in some estuaries. We have not tested this idea: the dispersive effect of local wind has not been quantified in Willapa, although Hickey et al. (2002) find that the site-to-site variation in mean along-channel velocity suggests a complex tidal residual field more than it does a wind-driven circulation. In the next section we verify the tidal-stirring scaling (3) by examining the dependence of empirical values of $K$ (defined as the "background" level of dispersion that persists in low-riverflow, unstratified conditions) on both $U_T$ and $b$.

\textit{b. Calculation of K}

The empirical relationship between gravitational circulation and riverflow in Willapa found by BHN02 can be written

$$\frac{u_0 \delta s}{Q a} \leq \mathcal{O}(1)$$

(4)

This result lets us use the river-flushing term $Q^{-1} a$, for which we have a continuous time series, as an upper limit on $u_0 \delta s$, for which we do not. Thus when $Q$ is low enough that the river-flushing salt flux is small compared with the time-change term, (1) becomes a simple diffusive balance

$$l_a \frac{\partial \bar{s}}{\partial t} = K \frac{\partial \bar{s}}{\partial x}$$

(5)

We thus define $K$, at a particular station, as the mean slope between the rate of change of salt storage and the axial salinity gradient in low-flow conditions.

We define "low flows" statistically, as those for which the river-flushing term is smaller than the std dev of the time-change term around its correlation with $\partial \bar{s}/\partial x$ (Fig. 3 below). At these flow levels, the bias in our estimate of $K$ due to neglected
gravitational circulation is, by definition, well-contained within the 95% confidence limits we place on $K$.

Results are shown in Figure 3 for all four stations within Stanley Channel. The data shown have been Butterworth-filtered with a cutoff frequency of $(10 \text{ d})^{-1}$: this averages over the mean propagation time for ocean signals from the mouth to the head at Naselle (Fig. 4 below). At Naselle, variance along both axes is small when river output is weak (Fig. 3a), and thus the estimate of $K$ there is poorly constrained and not different from zero at the 95% confidence level. It is, however, a stable result, $O(100 \text{ m}^2 \text{ s}^{-1})$ regardless of the choice of filter timescale or low-flow threshold.

At all stations the scatter of individual events around the long-term mean is often $O(1)$. This variance presumably results from a number of factors: the small but nonzero river-driven salt fluxes permitted under our definition of "low flows"; spring-neap variations in tidal stirring (sec. 3d); ocean-driven stratification changes and sea-level setup/setdown, which affect our estimate of the time-change term; lateral circulations driven by local wind; and non-Fickian tidal dispersion mechanisms (i.e., diffusive salt fluxes not proportional to $\partial \bar{S}/\partial x$: Ridderinkhof and Zimmerman 1992, Geyer and Signell 1992, McCarthy 1993). For many individual events during low-flow conditions, the net salt flux explained by the tidal-diffusion regression is smaller than the salt flux it does not explain. This may reflect baroclinic coupling between upwelling/downwelling signals at the coast and the gravitational circulation within the estuary, as suggested by Duxbury (1979) and observed in Willapa by Hickey et al. (2002).

Nevertheless, a constant diffusivity explains most of the variance in 10 d- and longer-scale salt fluxes everywhere except at the river mouth itself. (At W3, the regression in Fig. 3 has $r^2 = 0.71$; at Bay Center, $r^2 = 0.68$; at Oysterville, $r^2 = 0.61$; and at Naselle, $r^2 = 0.14$). In the subsections that follow we examine the physical meaning of the success of this very simple salt-flux parameterization, before comparing these diffusive salt fluxes to the gravitational circulation explicitly in section 4.
c. Signal penetration and propagation speed

A constant $K$ is only a satisfactory parameterization of up-estuary salt flux in low-flow conditions if it reproduces both the propagation rate and penetration distance observed for ocean signals. To derive predictions of these parameters from the axial profile of $K$ calculated above, we numerically integrated the differential form of (5)

$$a \frac{\partial \mathbf{s}}{\partial t} = \frac{\partial}{\partial x} \left( aK \frac{\partial \mathbf{s}}{\partial x} \right)$$

using a simple FTCS scheme, with data from W3 supplying the seaward boundary condition. Results are shown in Figure 4 for the low-flow period July-September 1999. Lag times of maximum correlation with W3, and amplitude of the correlated part of the signal (i.e., slope between salinity at each station and at W3) are shown for Bay Center and Oysterville. Naselle and the head of the channel, where signal penetration is weak and correlation poor ($r^2 = 0.2$), are omitted. The mean propagation rate for all stations and all forcing conditions is shown as well in Fig. 4a (this propagation rate is consistent with the rate found by Hickey et al. (2002) for lower-water-column stations during the 1995 low-riverflow period.) The numerical solution reproduces the empirical rate of propagation to within ~50%, and shows excellent agreement with the observed decline of signal amplitude (Fig. 4b). When single-frequency forcing is used in place of data from W3, equation (6) becomes the classical oscillatory boundary layer problem (Batchelor 1967, p. 353ff), and the numerical solution (not shown) reproduces the expected relationships between propagation rate, penetration distance, and forcing frequency.
d. Confirmation of the tidal residual scaling

$K$ is shown as a function of $U_T \cdot b$ in Figure 5. Since tidal velocity $U_T$ varies only fractionally over the length of Stanley Channel and channel width $b$ varies by a factor of 9, this correlation primarily tests the dependence of $K$ on $b$. We have defined $b$ as cross-sectional area divided by mean depth below MSL. The constant of proportionality $c_K$ varies between stations by only a small factor, and is close to the range of values assumed a priori in section 3a: it is $O(0.1)$, so that $K$ is, as predicted, similar to a tidal-residual velocity times the channel width at each station.

The scaling (3) predicts that $K$ increases linearly with tidal velocity, so that the rate of up-estuary salt flux is greater on spring tides than on neap tides. This prediction is contrary to the expectation for baroclinic exchange flows, which typically are retarded by spring tides because of increased vertical mixing (e.g., Park and Kuo 1996). Propagation rates between W3 and Oysterville are shown in Figure 6 for 12 events during the July–September 1999 period considered above. The trend with tidal height (a proxy for tidal velocity) is positive and close to linear, consistent with a tidally driven, rather than tidally inhibited, mechanism of exchange. The mean propagation rate for upwelling events in this small sample is greater than but not significantly different from the mean for downwelling events, as is consistent with the variable baroclinic coupling described by Hickey et al. (2002).

e. The tidal exchange ratio

We have shown that the diffusivities observed in Willapa during the low-flow-period are correlated with tidal motions as predicted by (3). As a stronger validation of our attribution of these diffusivities to tidal processes alone (rather than to, say, an
interaction between tidal and baroclinic circulations), we wish to associate these
diffusivity estimates with a net volume flux through the estuary mouth. If this volume
flux is tidally driven, it should not exceed one tidal prism per tidal cycle.

The ratio of the volume exchanged each tidal cycle between ocean and estuary to
the tidal prism is known as the *tidal exchange ratio* (Dyer 1973). For many applications,
the tidal exchange ratio, or the net volume flux itself (e.g. Austin 2002), may be a more
useful description of exchange than a diffusivity. In general we can relate these
parameters by

$$K \frac{\partial \bar{s}}{\partial x} = \frac{q}{\Delta} \Delta s_{in-out}$$  \hspace{1cm} (7)

where $q$ is the net volume flux and $\Delta s_{in-out}$ the net salinity difference between incoming
and outgoing flows. The tidal exchange ratio is then

$$\frac{q}{VT} \frac{TT}{VT} \hspace{1cm} (8)$$

where $T_T$ is the tidal period and $V_T$ the tidal prism volume. In Willapa, the total tidal
volume flux $V_T/T_T \approx 10,000$ m$^3$ s$^{-1}$.

The interpretation of $\Delta s_{in-out}$ depends on the exchange mechanism being
parameterized by $K$. For a gravitational circulation, $\Delta s_{in-out}$ is presumably related to
stratification. For our case of stirring by tidal residuals, it is more likely related to lateral
salinity gradients. We will guess that the axial salinity variation over one tidal excursion
is the largest variation upon which lateral shears can act: i.e. $\Delta s_{in-out} \leq \Delta s_{high-low}$
where $\Delta s_{high-low}$ is the salinity difference between high and low slack water at a particular
station. (This condition is consistent with the differential-advection mechanism
described by O’Donnell (1993) and others, and observed in Willapa by Hickey and
Banas (2002)). Rewriting $\partial \bar{s}/\partial x$ as $\Delta s_{high-low}$ divided by the tidal excursion $L_T$, (7) becomes

$$q \geq \frac{aK}{L_T}$$  \hspace{1cm} (9)
The rhs is roughly 6000 m$^3$ s$^{-1}$ at W3, implying (by (8)) a tidal exchange ratio for Willapa $\geq 0.6$. This value lies at the upper end of the values reported by Dyer (1973), consistent with Willapa’s large tidal range, wide mouth, and active, strongly advective coastal environment.

4. Gravitational and diffusive flushing regimes

In section 3 we determined $K$ by confining our attention to a forcing regime in which the salt balance is highly simplified. In this section we consider the role of tidal dispersion at higher riverflow levels, when all four terms in (1) are potentially important.

HR65, 66 introduced the parameter $\nu$, "the diffusive fraction of the total upstream salt flux," which is commonly used to describe the partitioning of salt flux between diffusive and baroclinic mechanisms. This parameter originally was defined in the context of a steady salt balance (2), however, and as we will show it becomes ambiguous and potentially misleading in an unsteady balance (1). Thus our goal in this section is twofold: to describe the partitioning of upstream salt flux in Willapa, and to use this case as an illustration of a very general theoretical concern.

We will proceed as follows. Section 4a discusses the generalization of $\nu$ to unsteady systems, and shows that while $\nu$ in the steady theory is one parameter that can be calculated three ways, in the unsteady theory these three ways become three distinct parameters that must be named individually. Sections 4b,c describe the partitioning of salt flux in Willapa in particular. Finally, sections 4d,e use these results from Willapa to demonstrate the independence of the three versions of $\nu$ defined in section 4a, and by extension the inconsistency of the predictions of the HR65 theory when applied to an unsteady estuary.
For reference, the parameters and salt balances described in this section are summarized in Table 1.

\( a. \) Defining \( \nu \) in steady and unsteady balances

In the steady balance (2), a single parameter is sufficient to describe the dominant balance among the three terms. This is the purpose served by \( \nu \), which HR65 write as

\[
K \frac{\partial \bar{s}}{\partial x} \frac{Q}{a} = (10)
\]

By (2), this is equivalent to

\[
\frac{K \partial \bar{s}}{\partial x} = u_e \delta s + K \frac{\partial \bar{s}}{\partial x} (11)
\]

which more literally matches the verbal definition of \( \nu \). Note that unless \( K \) is known independently (as it rarely is), neither (10) nor (11) can actually be used to compute \( \nu \). In practice (e.g., Bowden and Gilligan 1971, Oey 1984) one must use, explicitly or implicitly, a third rearrangement of the terms in (2)

\[
1 - \frac{u_e \delta s}{Q \bar{s}} = (12)
\]

This expression represents an imbalance between riverflow and gravitational circulation. It is directly related to the position of an estuary on the "Hansen and Rattray diagram," or stratification-circulation diagram, introduced by HR66: this diagram plots \( \delta s/\bar{s} \) against a quantity ~ \( u_e Q^{-1} a \) (Scott 1993), so that the product of the coordinates is very similar to one minus expression (12). (The exchange parameter \( \xi \) defined by BHN02 is one minus expression (12) identically.)
The expressions (10), (11), and (12) are equivalent in a steady balance but not in an unsteady one. The introduction of time-dependence adds another degree of freedom, which can be represented by the unsteadiness parameter $\psi$ introduced by BHN02, the ratio of time-change to river-flushing salt flux

$$\psi = \frac{l \frac{\partial \tilde{s}}{\partial t}}{\frac{Q}{a}}$$

(13)

When $\psi \neq 0$, river-flushing salt flux no longer represents the sum of up-estuary fluxes, and the balance between gravitational circulation and riverflow no longer constrains the partitioning of up-estuary fluxes. To distinguish (10), (11), and (12) we will name them individually: expression (11), since it reflects most literally the "diffusive fraction of up-estuary salt flux," will retain the symbol $\nu$; we will denote the original "steady" expression (10) as $\nu_s$; and we will refer to the "exchange" expression (12) as $\nu_e$.

The degree of discrepancy among $\nu$, $\nu_e$, and $\nu_s$ is $O(\psi)$:

$$\nu_s = (1 + \psi) \nu$$

$$\nu_e = (1 + \psi) \nu - \psi$$

(14a,b)

Thus even in quasi-steady systems, i.e., when $|\psi| = O(1)$, discrepancies can be large enough to be qualitatively misleading: to make either gravitational or non-gravitational exchange seem important or unimportant depending on which expression for $\nu$ one uses. Note that both estimates based on the steady theory can take on unphysical values ($\nu_s > 1, \nu_e < 0$) if $\psi$ is large.

To summarize: $\nu$, defined by expression (11), is literally "the diffusive fraction of up-estuary salt flux." Two other parameters $\nu_s$ and $\nu_e$, defined by (10) and (12), can be fairly conflated with $\nu$ in a steady system, but may differ from it even to order of magnitude in an unsteady one. This discrepancy is noteworthy because $\nu_s$ is traditionally given as the definition of $\nu$, and because only $\nu_e$, not $\nu$, can be associated directly with the Hansen and Rattray diagram. In an unsteady estuary, the relationship
between gravitational circulation and riverflow \( (v_e) \) is independent of the diffusive fraction of up-estuary salt flux \( (v) \), while in the HR65, 66 theory each process is fully constrained by the other.

To illustrate these conclusions, we return to the case of Willapa. We will start by presenting the relative roles of tidal and density-driven exchange in the most straightforward but least detailed terms, through a critical-stratification parameter, in sec. 4b; give a more detailed "climatology" of \( v \) in sec. 4c; and finally compare \( v \) with \( v_e \) and \( v_s \) directly in sec. 4d,e.

**b. River- and tide-controlled regimes**

The equipartition point between upstream salt fluxes

\[
\frac{u_e \delta s}{K \frac{\partial \delta s}{\partial x}} = 1 \quad (15)
\]

can be taken as the threshold between a diffusion-dominated, or "tide-controlled" regime, and a gravitational-circulation-dominated, or "river-controlled" regime. This threshold is equivalent to \( v = 0.5 \). To the extent that \( u_e \) is proportional to \( \frac{\partial \delta s}{\partial x} \) (HR65, Hickey et al. 2002, BHN02), the threshold (15) is also equivalent to a critical stratification

\[
\delta s_{crit} = \frac{K}{c_{u \times x}} \quad (16)
\]

where \( c_{u \times x} \) is the constant of proportionality

\[
u_e = c_{u \times x} \frac{\partial \delta s}{\partial x} \quad (17)
\]

The HR65 solution expresses \( c_{u \times x} \) in terms of channel depth and vertical diffusivity, but it may also be thought of simply as an empirical parameter.
Transcet observations of stratification relative to $\delta_{crit}$ are shown in Figure 7. Only the highest stratifications observed exceed it at W3, Bay Center, and Oysterville, although it is exceeded nearly half the time at Naselle. Most of the channel under most conditions thus appears to lie in the tide-dominated regime. This is not because stratification is consistently low, but rather because the threshold stratification is very high. At W3, $K$ is so large that density-driven exchange only exceeds "background" tidal diffusion when $\delta_s > 7$ psu.

Since $\delta_s > \delta_{crit}$ for most of the ~50 events observed, it appears that Willapa is tide-controlled ($\nu > 0.5$) under most, but not all, conditions. In the next subsection we examine the dependence of $\nu$ on river forcing with more precision.

c. A climatology of $\nu$

The upper limit of the empirical relation (4) between gravitational circulation and river-flushing is the case of "baroclinic balance": $\nu_e \approx 0$, or in the notation of BHN02, $\xi = 1$, or

$$\frac{Q}{a} \approx \bar{u} \delta_s$$

From this balance and the HR65 solution, BHN02 derive the scalings

$$\frac{\partial \bar{s}}{\partial x} \propto Q^{1/3}$$

$$\delta_s \propto Q^{2/3}$$

BHN02 also confirm (19a) observationally in Willapa, and find (19b) to hold as an upper limit on stratification.

We reintroduce these results here because (17) and (19) imply a relationship between $\nu$ and $Q$

$$\nu \propto \frac{1}{1 + const. \cdot Q^{2/3}}$$

(20)
This relationship is inconsistent with HR65, 66, in which (18) and (19) only hold for \( v = 0 \). Nevertheless, a functional fit to empirical values of \( v \) at each mooring station, of the form (20) but with the exponent on \( Q \) left free to vary, is consistent with the predicted value of 2/3 at the 95% confidence level. The top panels of Figure 8 show fits of the form (20) for each station. Here \( v \) has been calculated for each concurrent observation of \( \delta s \) from transect data and \( \partial \tilde{s}/\partial x \) from mooring data. The main panel of Fig. 8 gives a map of \( v \) as a function of axial position and riverflow level.

At Naselle, \( v \) depends sensitively on \( Q \) and takes on both very low (< 0.2) and very high (> 0.8) values over the typical annual range of riverflow. As one moves seaward, the seasonal range of \( v \) is increasingly muted, until at W3, the gravitational circulation does not reach the salt-flux equipartition point (\( v = 0.5 \)) even at the highest flows observed. The \( v > 0.8 \) regime, in which the gravitational circulation has only a second-order role in the salt balance, encompasses most of the volume of the estuary (Stanley Channel as far south as Oysterville) and riverflow levels up to the long-term median.

As in section 3, these results are averages over many events—here, an average by riverflow level—and so the river- and tide-controlled regimes we have identified describe seasonal-scale processes and the gross classification of the estuary, not necessarily the progress of individual pulses of ocean water. Baroclinic dynamics in particular are likely to vary substantially between individual events, although the present dataset conflates this real variance around the power-law fits in Fig. 8 with tidal aliasing of the stratification record.
The independence of $\nu$ and $\nu_e$

We can now compare $\nu$, $\nu_s$, and $\nu_e$ directly. The baroclinic-balance scalings (19) suggest further scalings

\[ \nu_s \propto Q^{-2/3} \]
\[ \nu_e \propto Q^0 \]  
(21a,b)

These power laws, like (20), are consistent with fits to data at every station within 95% confidence limits. Results are shown in Figure 9 for Bay Center.

If $\nu$, $\nu_s$, and $\nu_e$ are interpreted strictly, then the fit curves in Fig. 9 are self-consistent and suggest the following pattern:

- At low flows, on average, the salt balance is strongly unsteady ($|\psi| >> 1$). Gravitational circulation is at most comparable to river-flushing ($\nu_e$ small), and tidal stirring dominates over both ($\nu_s >> 1$, $\nu \approx 1$). The unsteady diffusive balance (5) is thus the dominant balance in (1), and the baroclinic balance (18) is second-order.

- At high flows, on average, the salt balance is steady or quasi-steady ($|\psi| \leq 1$). Gravitational circulation is still comparable to river-flushing ($\nu_e$ small), but now both terms are the same order as tidal stirring ($\nu \approx 0.5$). Thus all four terms in (1) contribute to the first-order balance.

In the preceding summary, $\nu$ functions as its name suggests, as a descriptor of the role of diffusive salt fluxes, while $\nu_e$ serves as a descriptor of the gravitational-circulation/riverflow relationship. To the extent that $\nu_e$ is constant as predicted, $\nu_s$ and $\nu$ are redundant.

If instead $\nu_s$ and $\nu_e$ are interpreted as estimates of $\nu$ on the basis of their equivalence in the steady theory, then they prove inadequate to the task as predicted in section 4a. The $\nu_s$ estimate is unphysically large except at higher-than-median flows.
The $\nu_e$ estimate predicts that gravitational circulation dominates at all flow levels, even when $\nu$ itself is very close to 1. The degree of divergence between these estimates is roughly proportional to $\psi$, as predicted by (14). The steady estimates $\nu_s$ and $\nu_e$ correctly predict the dynamical classification of the estuary only when $\psi < O(1)$, at the highest flows.

e. Willapa on the Hansen and Rattray diagram, revisited

We have found that $\nu_e$ (based on observations of the baroclinic circulation) is a poor predictor of $\nu$, which we have calculated directly from the horizontal tidal diffusivity. The discrepancy between $\nu_e$ and $\nu$ appears to be a direct consequence of the unsteadiness of Willapa’s salt balance, and thus is likely to be a general result for unsteady systems.

We observed in section 4a that $\nu_e$ is closely associated with the Hansen and Rattray diagram. This diagram is constructed from measures of baroclinic processes alone ($Q, u_\psi, \delta s$) but is frequently used to diagnose the importance of nongravitational processes, through $\nu$. HR66 in fact show contours of $\nu$ in the stratification-circulation parameter space, which follow from the steady-state assumption that $\nu_e = \nu$. The result is a direct association between hydrography and dynamics: the farther a partially-mixed (“type 2”) estuary lies from the well-mixed (“type 1”) region on the Hansen and Rattray diagram, the more the exchange flow is predicted to dominate over diffusion.

Data from Bay Center are shown on a Hansen-and-Rattray diagram in Figure 10. We have indicated observations for which $\nu > 0.8$ (i.e., for which baroclinic exchange is a second-order process) and also the region in which $\nu_e > 0.8$ (i.e. the region for which the HR66 theory predicts that $\nu > 0.8$). The $\nu_e$ region is defined by the axes and is fully general.
The $\nu_e$ and $\nu$ regions resemble each other in neither shape nor extent. The steady theory predicts that nongravitational processes dominate only when the baroclinic balance fails ($\nu_e \approx 1$), as, for example, when intrusions of oceanic freshwater erase stratification and weaken the axial salinity gradient (Hickey and Banas 2002, BHN02). Observations show that nongravitational processes dominate in Willapa under all except high-stratification conditions. (If the scalings (16) and (17) held exactly, contours of $\nu$ would be horizontal lines on the Hansen and Rattray diagram, simple stratification thresholds.) The points misclassified by HR66 (i.e. the $\nu > 0.8$ points outside the $\nu_e > 0.8$ region) encompass the majority of mid-to-late summer observations.

5. Discussion

a. Flushing dynamics of a Pacific Northwest coast estuary

In low-flow conditions, a simple diffusive model, in which $K$ varies spatially but does not change with riverflow or hydrography, reproduces the mean pattern of upstream propagation of ocean salinity fluctuations through Willapa Bay (Fig. 4). The rate and amplitude of the intrusion of individual fluctuations do vary around this mean within a small factor, as a result of spring-neap tidal variations (Fig. 6), baroclinic coupling (Hickey et al. 2002), and possibly wind-driven processes as well. Note also that we cannot generalize fully to ocean-estuary exchange of tracers other than salt: as Jay et al. (1997) note, "residence time" is a property of a tracer, not a basin. Our analysis of flushing mechanisms and rates applies to tracers well-correlated with salt, i.e., ocean-derived tracers whose replacement rates are much higher than their rates of gain or loss within the estuary. Whether this category includes the tracers of greatest biological interest, nutrients and biomass, most likely depends on season and, within the estuary, along-channel position. It is possible that during the onset of summer upwelling events, in which $\partial \bar{s}/\partial x$ is high but $\partial s$ too low for the gravitational circulation to move a
significant amount of salt, the baroclinic flow $u_e$ is still an important exchange mechanism for other tracers with different vertical distributions.

Nevertheless, the success in the long-term average of a density-independent model of salt intrusion (the scaling (3)) is itself surprising for an estuary with significant stratification. The stratification threshold at which gravitational circulation becomes important compared to diffusion (several psu, over most of the channel) is much higher here than often assumed for North American estuaries. Gross et al. (1999), for example, in a model study of South San Francisco Bay, find that even in nearly well-mixed conditions, it is necessary to include baroclinic exchange and the nonlinear effect of stratification on shear to correctly model the length of the salt intrusion. This may not be the case for an estuary like Willapa, with stronger tidal forcing.

Dynamical balances in low- and high-flow conditions were summarized in section 4d above. Comparison with Northern European estuaries, rather than with other North American estuaries, may be most fruitful. Dronkers and van de Kreeke (1986) found that the Volkerak, a small, macrotidal system, is flushed by nongravitational processes in its seaward reach but by baroclinic exchange near the head, similar to our conclusion for Willapa at moderate-to-high flows (Fig. 8). Simpson et al. (2001) found in the Conwy in Wales that for all except occasional high-riverflow events, the tidal upstream salt flux greatly exceeds the downstream river-driven flux, as we have found for Willapa during summer.

The factors that make lateral tidal stirring such an efficient exchange mechanism in Willapa appear to be shared by the other outer-coast estuaries of the Pacific Northwest. These include the large vertical tidal range, active inner-shelf environment, and broad, open intertidal areas (Emmett et al. 2000, Hickey and Banas 2002). The dependence of tidal diffusivity on basin morphology is not well-understood, although within Willapa $K$ does appear to have a close-to-linear dependence on channel width (Fig. 5) and tidal velocity (Fig. 6) as a simple scaling argument predicted (sec. 3a). We
speculate that the strength of tidal stirring on this coast may be increased by the fact that where East Coast estuaries have high and channelized intertidal salt marshes, Pacific Northwest estuaries generally have broad and relatively deep sand or mud flats (Emmett et al. 2000). The common East Coast marsh cordgrass *Spartina alterniflora* is not native on this coast but is invasive and currently spreading (Feist and Simenstad 2000): the southern reach of Stanley Channel, where $K$ is low, is already dominated by *Spartina* marshes. The connection between intertidal morphology and flushing dynamics should thus be a pressing concern on this coast.

*b. Steady theory and unsteady estuaries*

The gravitational circulation and tidal exchange appear to be only weakly coupled in Willapa. River-flushing of salt constitutes a limit on the gravitational circulation even during the wide range of conditions ($\nu > 0.8$, Fig. 8) in which neither is important to the salt balance as a whole. This weak coupling is not possible in a steady salt balance. According to the HR65 and HR66 diagnostics, a gravitational circulation in balance with riverflow ($\nu_e \approx 0$) is evidence that diffusive exchange is weak ($\nu \approx 0$). The example of Willapa shows that when external forcing is highly unsteady, this inference can be grossly misleading. It may nevertheless be internally consistent: even for Willapa Bay in late summer (which to first order functions like a riverless tidal embayment) it is possible to construct a plausible but inaccurate Hansen-and-Rattray-like salt-flux model by overlooking unsteadiness and diffusive salt fluxes simultaneously.

To demonstrate this, consider an alternative inquiry into Willapa in which we make the assumption of steadiness ab initio following HR65, never calculate $\partial \tilde{s}/\partial t$, and begin from (2) rather than (1). Our method for calculating $K$ from section 3 is unavailable in this framework, and so $K$ must be inferred from the other terms in (2). The baroclinic-balance analysis of BHN02, which proceeds unchanged, concludes that
in the long-term mean, \( \xi \approx 1 \) and \( \nu \) (calculated as \( \nu_r \)) is \( O(0.1) \). Expression (10) then suggests that \( K = O(100 \text{ m}^2 \text{s}^{-1}) \) near the mouth. The low value of \( \nu \) implies that density-driven exchange dominates at all riverflow levels, and that the residence time of the estuary (inversely proportional to the rhs of (2)) is \( \sim 10 \) times longer in summer than in winter. None of these inferences are correct: we found that \( K = O(1000 \text{ m}^2 \text{s}^{-1}) \) near the mouth and that the residence time, set by \( K \) rather than the river-driven fluxes, changes only fractionally between summer and winter.

Note that the conclusion to which the faulty steady-state assumption leads makes Willapa seem much more dynamically similar to other estuaries on the U.S. West Coast (Tomales Bay, Smith et al. 1991, Largier et al. 1996) and to canonical examples of partially mixed estuaries (the James River, Pritchard 1952) than it is. The simplicity of this pathway from familiar assumption to familiar, self-consistent and wrong conclusion is provocative. We hope that the example of Willapa Bay spurs reexamination of estuaries in which unsteady and density-independent exchange processes have been assumed, rather than observed, to be unimportant.

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UNSTEADY CASE ($\psi \neq 0$)

\[
I_a \frac{\partial \bar{s}}{\partial t} + \frac{Q_s}{a} = u_e \delta s + K \frac{\partial \bar{s}}{\partial x}
\]

salt balance

\[
\psi + 1 = \xi + \nu_s
\]

scaled salt balance

\[
\psi = \frac{I_a}{Q_s} \quad \xi = \frac{u_e \delta s}{Q_s} \quad \nu_s = \frac{K \frac{\partial \bar{s}}{\partial x}}{Q_s} \quad \nu_e = 1 - \frac{u_e \delta s}{Q_s} \quad \nu = \frac{K \frac{\partial \bar{s}}{\partial x}}{u_e \delta s + K \frac{\partial \bar{s}}{\partial x}} \quad \equiv 1 - \frac{\xi}{\xi}
\]

in the steady case, $\nu_s = \nu_e = \nu$

Table 1. Summary of salt flux ratios and balances referred to in the text.
Figure 1. Map of Willapa Bay, with mooring locations, study area, and transect route indicated.

Figure 2. Methods of calculation for the quantities in the subtidal, volume-integrated salt balance (eqn. (1)).

Figure 3. (a) Time change salt flux vs axial salinity gradient at each Stanley Channel station, in low-flow conditions. The upper riverflow ($Q$) threshold used is given for each station. The slope of the regressions (solid lines) is equal to diffusivity $K$; dashed lines show 95% confidence limits on the slope. (b) Diffusivities from (a) shown as an along-channel profile.

Figure 4. (a) Lag times relative to W3 for detrended salinity data from Jul-Sep 1999 (dots), and a numerical solution to (6) for the same time period (solid lines). The mean propagation rate for all five stations over the full three-year record (dashed line), ± 1 std dev (shaded region), is also shown. (b) Relative amplitude of the part of the salinity signal correlated with W3 for the summer 1999 data and numerical solution shown in (a).

Figure 5. Diffusivities from Fig. 3 compared to the product of tidal velocity and channel width. The shaded region corresponds to the prediction (sec. 3a) that the constant of proportionality $c_K \approx 0.1–0.2$.

Figure 6. (a) Detrended subtidal salinity at W3 and Oysterville, Jul-Sep 1999. (b) Signal propagation rate, inferred from the lagged correlation between W3 and Oysterville for individual maxima and minima in the record shown in (a), regressed to tidal amplitude.
Figure 7. Frequency of stratification observations at four stations, relative to the critical stratification level (indicated by $\nu = 0.5$).

Figure 8. Diffusive fraction of up-estuary salt flux $\nu$ as a function of axial position and riverflow level. The frequency distribution of riverflow is shown to the right. Contours are calculated using power-law fits, based on eqn. (20), to observations at each station (top panels).

Figure 9. Power-law fits to $\nu$, $\nu_s$, $\nu_e$, and $\psi$ as functions of riverflow at Bay Center. Transect data used to fit $\nu_s$ and $\nu_e$ is also shown; $\nu$ data used is shown in Fig. 8, and $\psi$ data by BHN02.

Figure 10. Stratification-circulation diagram following HR66 for transect observations at Bay Center. Open circles indicate $\nu < 0.8$, filled circles $\nu > 0.8$. Where multiple observations were made in $< 3$ d, an arrow indicates the path followed in parameter space. The region for which $\nu_e > 0.8$ is shaded.
Figure 1.
\[
l \frac{\partial \tilde{s}}{\partial t} + \frac{Q}{a} \tilde{s} = u's' + K \frac{\partial \tilde{s}}{\partial x}
\]

**Figure 2.**

- **mean upstream salinity:** volume-weighted sum of upstream mooring salinities
- **riverflow:** regression to USGS gauges
- **cross-sectional mean salinity:** mooring salinities used without adjustment
- **horizontal diffusivity:** slope between time-change term and \( \partial \tilde{s}/\partial x \) in low-riverflow conditions (see sec. 3)

- **CHANGE IN SALT STORAGE**
- **RIVER FLUSHING**
- **BAROCLINIC EXCHANGE**
- **DIFFUSIVE STIRRING**

- **length scale =** upstream volume / cross-sectional area \( a \)
- **cross-sectional area below MSL**
- **exchange-flow scale velocity:** regression between \( \partial \tilde{s}/\partial x \) and point velocities in the lower water column
- **axial salinity gradient:** at each station, salinity difference high-slack – low-slack, divided by the tidal excursion, low-pass filtered
- **stratification:** vertical salinity range from transect data (tidally aliased)
Figure 3.
Figure 4.
Figure 5.
Figure 6.
Figure 7.
Figure 8.
Figure 9.
Figure 10.

Bay Center data:

- \( \nu < 0.8 \)
- \( \nu > 0.8 \) — (tide-controlled: empirical)
- \( \gamma \) transects < 3 d apart
- \( \nu_e > 0.8 \) — (tide-controlled: HR66 prediction)

\[ \frac{u_{\text{surface}}}{u} \approx \frac{3}{2} + \frac{u_e a}{Q} \]